CELLULAR AUTOMATA BASED MODELING OF THE FORMATION AND EVOLUTION OF SOCIAL NETWORKS: A CASE IN DENTISTRY

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Abstract: The stability and evolution of networks is a research area that is not well explored. There is an inadequate focus to the partner characteristics and to the environment influence over the evolution process. This study analysed social network formation in its dynamic aspect through agent-based modeling, using cellular automata. Relationships and decisions were modeled. Along the interactions the emergence of consensus in the network could be observed and the results show that the more impulsive the individuals in a network, the stronger will be the ties among them. Convergence of partner selection criteria could also be noticed. Additionally, a structural hole could be shown to have a local influence on how it moves an agent away from the network. This work waves positively towards using cellular automata in social (in the case, business) networks modeling, in spite of their well-known limitations for these kinds of problems.

1 Introduction

The main objective of this study was to analyze the use of cellular automata (CAs) in modeling the dynamic process of partner selection in business network formation in the dentistry context, starting from a set of characteristics of potential partners; a secondary goal was to verify the influence of interactions among the dentists in the network dynamics. Specifically, first we look at which variables are involved in how dentists get together with colleagues of distinct specialties so as to form a social network that allows, for instance, the redirection of a patient with special needs, beyond a particular dentist's expertise, to another dentist of the network, so that the patient can be better served. At the same we analyze how dentists make their choices in building up a social network. Then, a CA-based model is developed to explain some aspects of the network creation and overall phenomenology.

Dissemination and use of information in a social system can be compared to a complex adaptive system with a large number of individuals that interact with each other, in a non trivial way, generating a visible collective behavior (Goldenberg, Libai & Muller, 2001; Granovetter, 1976; Hegselmann & Flache, 1998; Macy & Willer, 2002; Nagpal, 1999; Tesfatsion, 2005). *Agent-based modeling* (ABM) is a computational method that allows the creation, analysis and experiments with artificial societies composed by agents that interact in a local and non trivial way, constituting their own environment in an emergent fashion (Cederman, 2003; Epstein & Axtell, 1996; Macy & Willer, 2002; Mitchell, 1994; Nagpal, 1999).

The traditional approach based on phenomenological laws explains the combination between previous conditions and the result of the phenomena. In the last decade, social scientists began to develop a different approach to the explanation, based on causal mechanisms instead of laws (Cederman, 2003; Epstein & Axtell, 1996; Macy & Willer, 2002; Sawyer, 2004). With this, models of emergent computation, such as the one explored herein, has had an increasing role.

Macy & Willer (2002) argues that some sociologists do not completely appreciate computational methods and relational modeling as tools for theoretical research. For the authors, ABM basically differs from prior use of computation in sociology because it considers interactions, instead of simply proposing algorithms and equations to represent behavioral processes.

ABMs or *artificial societies* are based upon four premises. Agents are autonomous, interdependent, follow simple rules, are adaptive and consider the past (Sawyer, 2004). ABM relies on a set of computational agents (with internal states, behavior rules and parallel operation) and their environmental specification; the communication among them is made by rules, specifying a network of connectivity that is activated through the agents' interaction and observing the emergent macro behaviors (Epstein & Axtell, 1996).

The next session reviews some cellular automata concepts, which is followed by a presentation of the research methods employed. Subsequently, the results obtained are discussed, and then conclusions are drawn.

2 Background: Cellular Automata

Cellular automata (CAs) are fully discrete, complex systems that possess both a dynamic and a computational nature. They consist of a grid-like regular lattice of cells, and a state transition rule (Wolfram, 2002). The cells in the lattice have an identical pattern of local connections to other cells, and are subjected to some boundary condition, usually periodic. Each cell can take on one of a discrete set of possible states, and the neighbourhood of a cell is defined as the cell, together with the others that are connected to it.

The state transition rule yields the next state for each cell, as a function of its neighbourhood, and, at each time step, all cells synchronously have their states updated. In computational terms, a cellular automaton is, therefore, an array of finite automata, where the state of each automaton depends on the state of its neighbours.

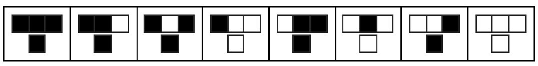

Figure.1 Rule 234 and the state transitions that define it.

For *elementary cellular automata* (ECA), that is, CAs where the neighbours are located only at the right and left of the centre cell, the size m of the neighbourhood is usually written as $m=2r+1$, where r is called the radius of the automaton. In the case of binary-state CAs, the transition rule is given by a state transition table, which lists each possible neighbourhood together with its output bit, that is, the updated value for the state of the central cell in the neighbourhood. Figure 1 gives an example, with rule 234 of the *elementary space* – the set of one-dimensional cellular automata rules with 2 states per cell and radius 1 – in which black cells represent state 1, and white cells represent 0. The rule denomination as 234 comes from the decimal number corresponding to the binary number that is formed from its rule table, from neighbourhood 111…1 on the left-hand side, as shown in the figure; in fact, such a naming scheme is widely used in the literature, for any radius, so that here we preserve it.

3 Modeling

3.1 Variables considered

A number of concepts discussed in the literature (for instance, Granovetter, 1973; Frenzen & Nakamoto, 1993; Khana, Gulati & Nohria, 1998; Herrnstein, 1990) led us to elaborate the hypotheses of the present study:

H1 A structural hole in the social network has local influence. *Structural hole* (Burt, 1997) is an imperfection in an network structure, due to a lack of information transmission to some of the agents of the network.

H2 The existence of an individual that acts as a structural hole in the network increases this individual distance to the network.

H3 The greater the convergence of partner selection criteria, the stronger will be the ties among the individuals.

H4 The larger the distance between individual and network rationale the larger the tendency to the individual to move away from the network.

H5 The stronger the tie between two actors, the greater is the impulsiveness in their decision making.

H6 The lesser the impulsiveness of an actor, the further away from the network the actor will be kept.

In pursuing the latter, the strength of ties among partners was measured herein through the product of the number of clients indicated to the partner during a week and for how long these indications occurred added by the number of clients received in a week and for how long the clients have been received. This way of measuring the construct is coherent with Granovetter (1973).

The business scope was measured herein by summing the amount of specialties offered, number of employees, number of work days in the week, and number of clients treated in a day. Additionally, network scope was measured by the amount of specialties involved in the relationship, in both directions.

In respect to partner selection, firms tend to make partner selection based on easily observable technical criteria, ignoring or underestimating personal compatibility criteria, important in initial stages of network formation and stabilisation. Some criteria were chosen to be tested in our network modeling according to some authors, listed below with their corresponding choice criteria:

- Partner's reputation choice was based, for instance, on Geringer (1991). This construct was measured following the idea of characteristics that define the reputation, which are: strategy quality, products/services quality, management quality, market orientation, innovation and financial strength.
- Partner's proximity choice was based on Geringer (1991).
- Expected partner's quality was measured by the technical knowledge, physical facilities and employees appearance, equipments, attention provided to the client, employees courtesy and availability to helping the client (Arrègle *et al.*, 2003; Geringer, 1991).
- Financial conditions offered by the partner to the client were defined as financial options and charges (Geringer, 1991).
- Resources complementary choice was based on Geringer (1991) and Hamel, Doz & Prahalad (1989).

Impulsiveness is considered a moderating variable, since it intervenes in the decision making process. The variables used to measure the construct – namely, the time spent to analyze the situation, level of emotion in the decision, degree of risk aversion, planning time and degree of qualitative and quantitative analysis of the benefits generated by the network participation – were all derived, for instance, from Doz & Hamel (1998).

Leadership is a variable that has influence over the network profile and probably moderates the individuals decision making process. It was included in the research but only the transformational characteristics were included. It was measured through the perception on the leadership characteristics of the partner.

The structural hole and its local effect will be measured evaluating the distances among the network participants, i.e., the differences among their partner selection characteristics along each one of the interactions.

3.2 Data collection

The sample was composed by dental offices located in São Paulo city, Brazil, where, according to the Regional Council of Dentistry database (CROSP) there are 17,571 dentists.

The sampling process followed a snow-ball type, i.e., starting with CROSP's database and then using the respondents indications. More specifically, invitations (for questionnaire answering) were initially sent to a randomly selected sample from the database, composed by 2200 letters and 960 emails. After that, the respondents were thanked by writing, where the opportunity was used to request from them the indication of potential new respondents. This procedure was repeated until a useful sample was reached, in the case 313 dentists; from these, 240 came from those directly contacted and 73 from the respondents' indications. Four questionnaires were excluded due to an excess of missing values, as they might compromise the modeling and statistical analysis. Eighteen questionnaires had a small amount of missing values, and were simply filled in by the valid sample average. The option for accepting incomplete questionnaires was appropriate, as it prevented the respondents from giving up the answering. Furthermore, four outliers were identified and removed from the modeling. In order to check whether their absence was relevant or not, the model was also run with them.

As an instrument to data collection, the questionnaire was hosted in a website. Also, a Likert scale from 1 to 6 was used, so as to avoid a neutral positioning of the respondent (Kerlinger & Lee, 2000). Before the actual research a pre-test was carried out with 30 individuals, which helped to improve the questionnaire, but the pre-test respondents were not considered in the final sample.

3.3 Data processing

After preparing the data, a factorial analysis, limited to 10 factors and Varimax rotation, was run and the principal components extracted.

The factors obtained from factorial analysis were orthogonal and had a Kaiser-Meyer Olkin sample adequacy measure of 0.587, and the result for the Bartlett Sfericity Test of 1865.52 significant at 0.000, meaning that factorial analysis was viable. Only variables with loads greater than 0.400 were selected. The outcome of the factor analysis served as variables to the modeling phase; they are listed below, together with the attributes they comprise:

1. *Leadership*: partner interest in the relationship, optimism of the partner regarding the future, partner that searches for alternatives, individual attention of the partner in the relationship and the perception that the network increased the business strength.
2. *Quality:* equipments, employees appearance, facilities, customisation of service and technology.

3. *Reputation*: ability, honesty and innovation.
4. *Customer care*: proximity, flexibility, financial conditions.
5. *Propensity to collaborate*: network scope, market saturation, market competition and risk aversion.
6. *Impulsiveness*: indication by a strong tie, planning horizon, emotion in the decision, persistence and inversely related to invested time in the decision making.
7. *Network utility*: calculation of benefits, contribution equivalence, dependence and inversely related to conflict and learning.
8. *Degree of external segregation*: importance given by a weak tie indication, elitism and hardness of the partner.
9. *Decision importance*: involvement and quantification of risks prior to the partnering.
10. *Decision value*: egotism, complementarity of resources, costs and risks of Dentistry practicing.

3.4 Implementation

Once we had the 10 factors, data was classified following the leadership factor, so that the individual that identifies the partner the most as a leader became the *pivot*, that is, the reference in the network organisation. The remaining individuals were placed to the right of the pivot, in decreasing order of the value of their leadership factor, so that the individual that identifies itself the least with the leader ends up at the left-hand side of the pivot, since the model has periodic boundary.

The different scales of factor values were then transformed into a single scale, from 0 to 255, and, for every factor, a distance measure was computed (D_{i0}) between every (i^{th}) individual and the pivot, according to the individual's factor values at issue,

by means of the expression $D_{i0} = \sum_{n=1}^{10} (X_{ni} - X_{n0})$,

where X_{ni} is the n^{th} factor X of the i^{th}-individual and X_{n0} is the n^{th} factor X of the pivot.

Then, a threshold scale was used to transform the decimal factor values into binary representation. The threshold scale was composed of classes and their number was defined by Sturge's rule $C=1+3.332*Log_{10}N$, where N is the sample size, namely, 305 individuals. Sturge's rule application led to 9 classes, defined by intervals of 28.333.

Each factor value was then converted to 9-bit-long binary number, following the threshold scale. Each binary number corresponding to a given factor value was created by associating bit 1 to the position corresponding to the interval containing the value, and 0 to the other positions, so that the resulting binary number would have just a single 1-bit at the position corresponding to the interval containing the value at issue. For example, a factor value of 43.3 would have a value 1 in the second bit and 0 in all others. So, for each factor, nine ECAs are then created, each one of them consisting of a 305-bit-long binary numbers. The initial state of the ECAs were then obtained and the state of each ECA was independently updated. The dimension chosen for the neighbours, one at each side, was selected to preserve information quality. With such an approach a society connected through strong ties has thus been created, in accordance with Chwe (1999), who argues that strong ties are better in the creation of common knowledge, an essential feature to collective action.

In order to select the most appropriate rule to run the model, a rule was chosen so as to allow more than 10 iterations, convergence to a fixed point and not a cyclic attractor in less than 308 iterations, and such that its transient would not be too long. These conditions were determined due to the computational limitations and in order to make data analysis possible. Rules that converged too rapidly (in less than 10 iterations) were eliminated, as well as the fixed point and chaotic rules. We considered fixed point as the point in which all cellular automata converged to a steady state.

The selection criterion also considered the possibility of the rule admitting a *rationale*, in that the rule could be translated into a coherent action with the theory. In tune with this idea, the chosen rule was rule 234 (displayed below), out of the 256 possible ECA rules; in addition to meeting all requirements, it embeds the rationale that an individual's opinion follows the local majority, but if the individual does not consider the specific partner selection criteria as relevant the same way the most influent neighbour (i.e., the one in the leftmost position) does, that is, both of them have value 0 to that partner selection criteria, the individual changes its original opinion and turns it to the same as the least influent neighbour; Fig. 1 illustrates the rationale. The change from 0 to 1 means that the individual's opinion change and the individual start considering the partner selection criteria as relevant. If 0 remains 0 it means that the individual does not change its opinion and does not consider that partner selection criteria as relevant in its choice.

The stop criterion of the ECAs associated to each factor was determined when all corresponding 9 ECAs had reached a fixed point, or, otherwise, after 308 iterations. At that moment each column of the lattice, i.e., individual values, was converted back to a decimal number following the average class intervals that contained 1-bits; more precisely, every cell containing a 1 was given the average value of that specific class, and then all values of that individual were summed up. Then, the final distances relative to the pivot were measured just like initially.

4 Results

Professionals working in small dentistry organisations containing 3 to 5 employees answered the research questionnaire and the major percentage of answers (36%) came from general practitioners.

Analyzing the responses it is possible to notice that only few variables were not highly considered important as partner selection criteria. Among them are strength of ties, perception of partner elitism, and price charged by the partner. Respondents have a medium to short term perspective in their decision making process, and see their partners as leaders.

In the modeling phase each rule generated a different special positioning, thus confirming hypothesis **H4**, because according to the existing rationale in the network, entailed from the rule, the individual attributes and distances change.

Initial distances had great variation, in the range from -250 to 250, which represents the individual differences regarding different selection criteria. The reduction of distances after rule 234 application to a range of 0.08 shows that individuals get closer to each other, due to tie strengthening and an increase in the degree of similarity. There was an opinion convergence, represented by the reduction of distances among individuals, but at the micro-level they did not become exactly the same. Results are compatible with the Matching Law, since similar individuals strengthen the tie between them and are susceptible to constant behavior reinforcements, therefore supporting hypothesis **H3**.

Final distances showed that individual 249 was the one who modified the least its positioning, keeping himself away from the network. This individual was the least influenced by the network and increased its factor values 9 fold, while the network increased 9.3 times. Analyzing the initial variables, we noticed that individual 249 does not consider partner indication by a strong tie as important, had considered more the benefits prior to entering the network, perceives that the contribution of its partner in the relationship is not equivalent, and takes more time than the average to decide. So he is a less impulsive individual, very rational and with low cooperation propensity. He wants to maximize its utility, perceives little value in the decision of participating in the network and is refractory to strong ties reinforcement. So, participating in a network has a small weight in its decision, thus making him a weak tie. Hence, such a finding supports hypothesis **H6.**

The increase in tie strength caused an increase in impulsiveness and a greater cooperation propensity, because there was the same rationality in the network, so that individuals were deciding accordingly. Network utility increased, as well as the outcome value and its importance, because according to the Matching Law, individuals are subject to behaviour reinforcements. These facts confirm hypothesis **H5**.

Initial distances had great variation, in the range from -250 to 250, which represents the individual differences regarding different selection criteria. The reduction of distances after rule 234 application to a range of 0.08 shows that individuals get closer to each other due to tie strengthening and an increase in the degree of similarity. that is, everyone perceives that artificial network formation increases each business strength. The reputation factor was the only one for which there was a convergence in two clusters. The smaller one was composed in its extremity by individuals 33 e 173 and its size was 85% smaller than the other. In order to know who was the individual that attracted the others to the second cluster, both were analysed.

Individual 33 does not see its partner as a leader, differently from individual 173. So, individual 33 is farther from the pivot. The larger discrepancy in the factor values, considering the rest of the network, happened for individual 33, suggesting that he is the one that changes its opinion the most. We concluded that he is the source of influence that moves away the adjacent individuals, up to individual 173. Those adjacent to 33 and 173 do not receive the information because 33 alters the quality of the information transported. However, individuals from 173 and beyond converge to zero because the CA has periodic boundary, that is, the pivot interacts with distant network layers. The periodic boundary shows the importance of the manager to be in contact with employees in order to spread its influence to the different layers of the organisation and of the network.

In order to verify the influence of outliers that were removed from the model, we ran the model with them. The only noticed alteration was the increase in time convergence, that turned from 199 to 288 iterations. This result is coherent, since the total number of individuals is much larger relatively to the number of outliers, and as consequence the latter were not capable of altering the overall system configuration.

The insertion of a structural hole using rule 204 in the 150th individual, that is, an individual that did not change its opinions along the interactions, moved away individuals from the 45th to the 150th, who formed a cluster. This confirms that the information flow comes from the individuals far from the pivot influence. This happened because the 150th individual interrupted information flow. However, information flow is compensated by transitivity. That is the reason why there was only a local influence, therefore supporting hypotheses **H1** and **H2.**

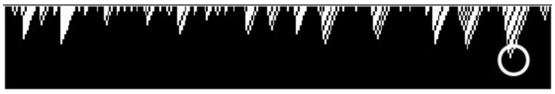

Figure 2: Quality factor CA evolution for the first CA. Initial condition is at the top, and time flows downward.

At iteration 26 there was an exponential threshold from where a characteristic convergence could be observed. In order to identify it properly we looked at the CA global state at that iteration. Fig 2 shows the first 70 iterations of the rule temporal evolution. The threshold is showed by the white circle at the tip of the next-to-last triangle. This threshold happens at the 26th iteration and represents the moment where the network is taken by the "rebels" (black areas), individuals that propagate their influence and represent the majority. It can be noticed that individuals far from the pivot join the network later (white triangles on the right whose size is bigger than the ones on the left). Iteration 26 determines the network threshold, where the majority of "rebels" take the network, a situation that is usually found at innovation adoption events. After 199 iterations, all the network is in consensus (black area), thus meaning that all network starts to consider quality as a relevant factor in partner selection.

5 CONCLUSIONS

The selected area of the sample used herein may not represent the general condition of the Dentistry universe used. Therefore, conclusions may not be general to all firms of the dental sector in São Paulo city. Clippings and options done to measure the constructs chosen as important to formation and development of networks were selected following the authors' objectives and do not include all partner selection decision criteria found in the literature. Nevertheless, many interesting observations could still be made, confirming the validity and strength of the model

Specifically, results show that a structural hole in the network increased the distance of adjacent individuals and its influence decreases in distant neighbours. Different distances also came from different network rationale, which means that environment influence prevailed over individual decision criteria. This happened because individuals with different decision criteria were attracted to the same position through the use of the same reasoning, suggesting that the environment exerts strong pressure over the individual and that decisions are contingent.

Along the interactions there was a convergence of opinions in decision criteria, which represents the consensus being formed among dentists. The distances decreased for all individuals, meaning that decision criteria similarity increased along the interactions. The increase of similarity among decision criteria caused a greater propensity to cooperate. There was an increase in tie strength among network participants, what caused an increase in their impulsiveness, compatible with the Matching Law.

Macy & Willer (2002) observe that many agent-based models treat social forms as behavioral interactions, not varying the topology and actor identities. The model explores this gap in the literature and considers topology, once it has special relevance in the modeling results (Sipper, 2004). The model also considers structural attributes of the agents, the same way Tesfatsion (2005) did, but in another circumstances. In tune with latter, actor identity is varied herein, insofar as the factors change along time, since individuals do interact with their neighbours and may acquire opinions that initially might have belonged to another. Topology varied according to discrepancies of the individuals' characteristics. and was measured by the distance among actors and a chosen individual (the so called *pivot*), in the same way as Burt (1976) and Gulati (1995).

All in all, this study contributes to theory insofar as it allowed to verify the potential use of cellular automata to understand formation and evolution of social networks, in tune with the enormous attention the topic has received recently, and in spite of well-known limitations of cellular automata use in these kinds of problems. Emergent phenomena was observed, and there were environment and rationale influences in network configuration. This fact suggests a dynamic partner selection decision criteria classification and allows to understand the formation of weak ties as well as the emergence of consensus in the network. This study also contributes to practice once it allows to understand the knowledge management in the network and the direction of information transmission.

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